

# Recent Advances on Dynamical Equations

*to celebrate Luisa Malaguti 60th birthday*

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## **Multiple anti-periodic solutions of implicit differential inclusions on tori**

Jan Andres

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A lower estimate of the number of anti-periodic solutions of implicit differential equations and inclusions on tori will be given. Our approach is based on the application of the topological essential fixed point theory, jointly with the Nielsen theory for multivalued maps.

## **A weak topology approach for differential inclusions in abstract spaces**

Irene Benedetti

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The existence of solutions for multivalued differential equations in abstract spaces is frequently studied with topological techniques based on fixed point theorems for a suitable solution operator. This requires strong compactness conditions, which are very hard to check in an infinite dimensional framework. In this talk will be presented an approach based on weak topology that weaken the compactness conditions usually required.

## **Nonlinear wavefronts arising in the modeling of collective movements**

Andrea Corli

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In this talk I shall focus on some recent results, obtained in collaboration with Luisa Malaguti and co-workers, about traveling waves for nonlinear scalar models of traffic flows or crowds dynamics. These results concern the existence or nonexistence of such waves, their smoothness and several other qualitative properties. Nonlinear diffusion plays a key role in these models, and the anomalous case of negative diffusivity, which indeed arises in a natural way in the modeling, will be discussed as well. A brief notice about discontinuous wavefronts shall also be given.

## **Global positive solutions vanishing at infinity for differential equations with indefinite weight**

Zuzana Dořlá

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This is a joint research with Serena Matucci, University of Florence. We investigate positive radial solutions for a nonlinear elliptic equation with  $p$ -Laplace operator and sign-changing weight, both in superlinear and sublinear case. We prove the existence of solutions  $u$ , which are globally defined and positive outside of a ball of radius  $R$ , satisfy fixed initial conditions  $u(R) = c > 0$ ,  $u'(R) = 0$  and tend to zero at infinity. Our method is based on a fixed point result for boundary value problems on noncompact intervals and on asymptotic properties of suitable auxiliary half-linear differential equations.

## **Some results on traveling fronts for strongly saturating diffusions**

Maurizio Garrione

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We present some recent results concerning heteroclinic traveling waves for reaction-diffusion models with strong saturation (for instance, in the case of a mean curvature-type diffusion). We first deal with the existence of traveling fronts, depending on the shape of the reaction term; we then insert a small parameter  $d$  in front of the diffusion, discussing the behavior of the fronts for  $d \rightarrow 0$  (vanishing diffusion limit). The results are mainly obtained through a shooting technique for a suitable first-order reduction of the original problem, together with a direct convergence analysis.

## **On Optimization of a Feedback Control System Governed by a Fractional-Order Differential Inclusion**

Valeri V. Obukhovskii

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For a feedback control system governed by a fractional order semilinear differential inclusion in a Banach space, we study the optimization of a given functional on a set of trajectories satisfying a given initial condition. The problem under consideration includes, as particular cases, the terminal control problem and a particular Bolza problem.

We deal also with the time-optimal problem which consists in finding of such a trajectory of the system which, starting from a given initial set attains a certain target set in the shortest time.

As example, we consider optimization problems for a time-fractional diffusion type system which include as a particular case the same problems for a controlled process of fractional heat transfer.

### **Bound sets approach to boundary value problems for differential equations and inclusions**

Martina Pavlačková

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Joint work with Jan Andres, Luisa Malaguti, Valentina Taddei. The first part of the talk deals with the historical development of a bound sets theory for the first order and the second order differential equations and inclusions. After historical overview, a recent result concerning the impulsive second order multivalued Dirichlet problem is studied. The method used for obtaining the existence and the localization of a impulsive solution is based on the combination of a fixed point index technique and the bound sets theory. Finally, the presented theoretical result is supplied by a numerical example.

### **Nonlinear differential models via topological methods**

Paola Rubbioni

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In 2016, the paper [1] appears, showing the results of a new approach to a model for biological invasions and diseases spread, carried on by means of topological methods. Therein, the parametric differential equation describing the process was transformed in an ordinary differential inclusion and the existence of controlled dynamics to the problems there considered - a periodic problem, a multipoint boundary problem, a mean value problem - was

proved, applying techniques of multivalued analysis and of degree theory for condensing operators in Hilbert spaces.

Later, following the ideas there introduced, several papers were published either on spatial nonlocal diffusion models and on differential models with distributed or functional delay.

This talk wants to illustrate the approach introduced in [1] and show some new results on delay models.

## References

- [1] Malaguti L., Rubbioni P., Nonsmooth feedback controls of nonlocal dispersal models, *Nonlinearity* (2016)

## **An approximation solvability method for semilinear equation in Banach spaces**

Valentina Taddei

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A new approximation solvability method is developed for the study of semilinear differential equations in Banach spaces without any compactness on the semigroup nor on the nonlinearity. The method is based on the Yosida approximations of the generator of the semigroup, on a continuation principle, and on the weak topology. The solutions are limits of functions with values in finitely dimensional spaces. An existence result is obtained for a wide class of nonlocal boundary value problems, including periodic, anti-periodic, weighted mean value and multipoint conditions; as well as for a non necessarily sublinear growth condition. By means of linear control terms, the controllability problem is also investigated. The final configuration is always achieved with a control with minimum norm. Applications to diffusion models are given.

## **Bound sets for second-order vector differential equations: a panorama of some results**

Fabio Zanolin

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In the study of boundary value problems associated with first and second-order vector differential equations, the “bound sets technique” is a powerful tool, in connection with topological methods, to prove the existence of solutions which remain inside a given set. This technique has been refined by Luisa Malaguti and collaborators in a series of important papers and applied to infinite-dimensional problems and differential inclusions, so that nowadays the set of available interesting results is too large to be summarized in a short lecture. In this talk, we survey a few classical results for the periodic problem and we also present some recent ones (obtained in collaboration with Guglielmo Feltrin) for the second-order quasi-linear system  $(\phi(u'))' + f(t, u) = 0$ .