

Abstracts invited talks



Nonlinear Elliptic PDEs in Ancona

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A LIONS TYPE RESULT FOR A LARGE CLASS OF ORLICZ–SOBOLEV SPACE AND
APPLICATIONS

Claudianor Alves, Universidade Federal de Campina Grande

In this paper we prove a Lions type result for a large class of Orlicz-Sobolev space that can be nonreflexive and use this result to show the existence of solution for a large class of quasilinear problem on a nonreflexive Orlicz-Sobolev space. *This a joint work with Marcos L.M. Carvalho.*

GROUND STATES FOR AN HARTREE-FOCK TYPE SYSTEM

Pietro d’Avenia, Politecnico di Bari

We introduce an Hartree-Fock type system made by two Schrödinger equations in presence of a Coulomb interacting term and a *cooperative* pure power and subcritical nonlinearity depending on a parameter $\beta \geq 0$.

We present some results about the existence of radial ground states solutions and their *semitriviality* or *vectoriality* covering the whole range $\beta \geq 0$.

Joint work in collaboration with Liliane Maia and Gaetano Siciliano.

SINGULARITY FORMATION FOR THE KELLER-SEGEL SYSTEM IN THE PLANE

Manuel del Pino, University of Bath

The classical model for chemotaxis is the planar Keller-Segel system

$$u_t = \Delta u - \nabla \cdot (u \nabla v), \quad v(\cdot, t) = \frac{1}{2\pi} \log \frac{1}{|\cdot|} * u(\cdot, t)$$

in $\mathbb{R}^2 \times (0, \infty)$. Blow-up of finite mass solutions is expected to take place by aggregation, which is a concentration of bubbling type, common to many geometric flows. We build with precise profiles solutions in the critical-mass case 8π , in which blow-up in infinite time takes place. We establish stability of the phenomenon detected under arbitrary mass-preserving small perturbations and present new constructions in the finite time blow-up scenario.

THE EVOLUTION OF ATOMIC DISLOCATIONS IN CRYSTALS

Serena Dipierro, University of Western Australia

We consider an evolution equation motivated by the Peierls-Nabarro model and we discuss its mesoscopic and macroscopic limits. The model deals with atomic dislocations in crystals and we discuss how the dislocation function has the tendency to concentrate at single points of the crystal, where the size of the slip coincides with the

natural periodicity of the medium. These dislocation points evolve according to the external stress and an interior potential, which can be either repulsive or attractive, depending on the relative orientations of the dislocations. For opposite orientations, collisions occur, after which the system relaxes exponentially fast.

WHICH MAGNETIC FIELDS SUPPORT A ZERO MODE?

Rupert L. Frank, California Institute of Technology

Motivated by the question from mathematical physics about the size of magnetic fields that support zero modes for the three dimensional Dirac equation, we study a certain conformally invariant spinor equation. We state some conjectures and present results in their support. Those concern, in particular, two novel Sobolev inequalities for spinors and vector fields. The talk is based on joint work with Michael Loss.

PRESCRIBED NORM SOLUTIONS OF SCHRÖDINGER EQUATIONS WITH MIXED POWER NONLINEARITIES

Louis Jeanjean, University of Bourgogne

In this talk, I will present some recent results concerning the existence of prescribed norm solutions in problems where the associated nonlinearity is the sum of two powers, one which is mass-subcritical and one mass-supercritical. This leads to consider a constrained variational problem presenting a so-called convex-concave geometry. The issues of existence, multiplicity and orbital stability of solutions will be addressed with a special emphasize on the cases where the mass-supercritical power is Sobolev critical. *The content of this talk is based on some jointed work with J. Jendrej (Paris) T. T. Le (Besançon) and N. Visciglia (Pisa).*

LOCAL MINIMA FOR CONSTRAINED MINIMIZATION PROBLEMS

Giovanni Molica Bisci, Università degli Studi di Urbino Carlo Bo

In the last years, local minima for constrained minimization problems have attracted several outstanding mathematicians and the interest towards this kind of problems has grown more and more, not only for their intriguing analytical structure, but also in view of their applications in a wide range of contexts. Motivated by this wide interest in the literature, the leading purpose of this talk is to present some recent results on elliptic equations, mainly related to a wide class of functionals defined through multiple integrals of Calculus of Variations. Applications to quasilinear boundary value problems will be presented and some open problems briefly discussed; see [1] and [2, Chapter 8] for related topics.

REFERENCES

- [1] G. MOLICA BISCI, *Local minima for some functionals in the Calculus of Variations*, submitted for publication (2021), 1–53.
- [2] G. MOLICA BISCI AND P. PUCCI, *Nonlinear Problems with Lack of Compactness*, De Gruyter Series in Nonlinear Analysis and Applications **36** (2021), i+vii, 1–266.

SOLUTIONS OF THE INCOMPRESSIBLE EULER EQUATIONS WITH CONCENTRATED VORTICITY

Monica Musso, University of Bath

I will discuss solutions to the incompressible Euler equation in two dimensions with vorticity close to a finite sum of Dirac deltas (vortices). The law of motion of the vortices was known formally for a long time and proved rigorously by Marchioro-Pulvirenti. In collaboration with Juan Davila (U. Bath), Manuel del Pino (U. Bath), and Juncheng Wei (UBC) we have a different point of view, which allows a very precise description of the solution near the vortices. Our construction can be generalized to other situations, such as the construction of leapfrogging vortex rings of the 3D incompressible Euler equations.

NON-LOCAL ODES IN CONFORMAL GEOMETRY

Maria del Mar Gonzalez Noguera, Universidad Autonoma de Madrid

When one looks for radial solutions of an equation with fractional Laplacian, it is not generally possible to use classical ODE methods. If such equation has some conformal invariances, one may rewrite it in Emden-Fowler (cylindrical) coordinates and use the properties of the conformal fractional Laplacian on the cylinder. Then it is possible to apply complex variable methods in order to obtain existence and regularity for such equation. A particular application is the study of fractional Laplacian equations with drift and a critical Hardy potential (joint work with H. Chan, M. Fontelos and J. Wei).

SINGULAR AND DOUBLE-PHASE PROBLEMS: NEW RESULTS AND SOME PERSPECTIVES

Vicențiu D. Rădulescu, University of Craiova and Institute of Mathematics of the Romanian Academy

In this talk, I shall report on some joint works in collaboration with P. Pucci (Perugia), N. Papageorgiou (Athens) and D. Repovš (Ljubljana). I shall be concerned with several types of singular or multi-phase phenomena arising in the theory of nonlinear elliptic equations. I will describe some striking results related to the existence of blow-boundary solutions, the generalized maximum principle, and a discontinuity

property of the spectrum for double-phase problems. We establish that no monotonicity hypotheses are necessary in the statement of some classical theorems. Several open problems are raised in the final part of this lecture.

LACK OF COMPACTNESS, SYMMETRIES AND A FLOWER-SHAPE GEOMETRY

Raffaella Servadei, Università degli Studi di Urbino Carlo Bo

Several important problems arising in many research fields, such as physics and differential geometry, lead to consider semilinear variational elliptic equations defined on unbounded domains of the Euclidean space and a great deal of work has been devoted to their study. From the mathematical point of view, probably the main interest relies on the fact that often the tools of nonlinear functional analysis, based on compactness arguments, cannot be used, at least in a straightforward way, and some new techniques have to be developed.

In a joint paper with Giuseppe Devillanova (Politecnico di Bari) and Giovanni Molica Bisci (Urbino) we introduce a group theoretical scheme, raised in the study of problems which are invariant with respect to the action of orthogonal subgroups, to show the existence of multiple solutions distinguished by their different symmetry properties.

Aim of the talk is to present this construction, called flower-shape geometry, and to show its applications to the study of nonlinear problems set in strip-like domains.

SYMBOLIC DYNAMICS AND ATOM DISLOCATIONS IN CRYSTALS

Enrico Valdinoci, University of Western Australia

We consider a nonlocal equation driven by a perturbed periodic potential. We construct multibump solutions that connect one integer point to another one in a prescribed way. In particular, heteroclinic, homoclinic and chaotic trajectories are constructed. This result regarding symbolic dynamics in a fractional framework is part of a study of the Peierls-Nabarro model for crystal dislocations: the methods of dynamical systems are exploited here to construct complex spatial configurations of the atom inside the crystal.