

# DOUBLE PHASE PROBLEMS: A LOOK AT THE PAST, AND A LOOK AT THE FUTURE.

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In the 80s Zhikov introduced several anisotropic model functionals from the Calculus of Variations, involving nonautonomous integrands, and produced several examples of Lavrentiev phenomenon. Many years later, I observed that one of these was particularly useful to produce yet further examples, showing the optimality of certainly higher integrability results I obtained with Esposito and Leonetti, and related to the general  $(p, q)$ -growth conditions introduced by Marcellini. That particular example always intrigued me, so that, again years later, jointly with Paolo Baroni and Maria Colombo, I decided to deepen the study of these functionals and in fact I also found a name for such a family, that is the one of double phase functionals. Since then, these functionals, and this terminology, has become very popular. In my talk I will give a summary of basic and recent results on such models, obtained with Cristiana De Filippis, and I will try to highlight some future directions.

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