ERRATA OF THE BOOK

(1) At pag. 10, replace
$$\mathcal{L}^p_\alpha = G_\alpha(L^p(\mathbb{R}^N)) = \{u : u = G_\alpha * f,\ f \in L^p(\mathbb{R}^N)\}$$
by
$$\mathcal{L}^p_\alpha = \mathcal{J}_\alpha(L^p(\mathbb{R}^N)) = \{u : u = \mathcal{J}_\alpha f,\ f \in L^p(\mathbb{R}^N)\},$$
and replace
$$\|u\|_{\mathcal{L}^p_\alpha} = \|f\|_{L^p(\mathbb{R}^N)} \text{ if } u = G_\alpha * f,$$
by
$$\|u\|_{\mathcal{L}^p_\alpha} = \|f\|_{L^p(\mathbb{R}^N)} \text{ if } u = \mathcal{J}_\alpha f.$$

(2) At pag. 110, formula (4.2.1), replace
$$\frac{|u(x)-u(y)|^2}{|x-y|^{N+2s}}$$
by
$$\frac{|u(x)-u(y)|^2}{|x-y|^{\frac{N+2s}{2}}}.$$

(3) At pag. 132, to obtain the exponential estimate for v we need to replace "$|x| \geq 2R$" by "$|x| \geq R' = \max\{\frac{rR}{R-1}, 2R\}$". In this way we have
$$|x-y| \geq |x| - |y| \geq |x| - r \geq \frac{|x|}{R} \geq 2 \quad \text{for all } |x| \geq R'.$$
(the inequality given in the book works for $|x| \geq 2R$ if we assume $r = 1$)

(4) At pag. 216, second line from above, replace "$d_\mu =$" by "$d_\mu + o_n(1) =$".

(5) At pag. 223, in the statement of Proposition 6.3.14, it is needed to add "$\mathcal{J}'_\varepsilon(u_n) \to 0$".

(6) At pag. 231, in the proof of Theorem 6.3.22, replace the sentences "On the other hand, let us choose $h(\varepsilon) > 0$ such that....we infer that Ψ_ε satisfies that Palais-Smale condition in $\tilde{\mathbb{S}}_\varepsilon$" by "On the other hand, let us choose $h(\varepsilon) > 0$ such that $h(\varepsilon) \to 0$ as $\varepsilon \to 0$ and $d_{V_0} + h(\varepsilon)$ is not a critical level for Ψ_ε. For $\varepsilon > 0$ small enough, we can argue as in the proof of Proposition 6.3.14 to deduce that Ψ_ε satisfies the Palais-Smale condition in $\tilde{\mathbb{S}}_\varepsilon$."
We note that one can prove the following result: "Let $(w_n) \subset \mathbb{S}_\varepsilon$ be such that $\Psi_\varepsilon(w_n) \to c$ and $\Psi'_\varepsilon(w_n) \to 0$, where $c < d_{V_\infty}$ if $V_\infty < \infty$ and $c \in \mathbb{R}$ if $V_\infty = \infty$. Then (w_n) has a subsequence that converges strongly in $\mathcal{H}_\varepsilon$." The proof goes as follows. Let $u_n = m_\varepsilon(w_n)$. It follows from Lemmas 6.3.2 and 6.3.3 that $\mathcal{J}_\varepsilon(u_n) \to c$, $\mathcal{J}'_\varepsilon(u_n) \to 0$ and $\langle \mathcal{J}'_\varepsilon(u_n), u_n\rangle = 0$. From Proposition 6.3.14, we deduce that $u_n \to u$ in $\mathcal{H}_\varepsilon$ for some $u \in \mathcal{H}_\varepsilon$. Clearly, $u \in \mathcal{N}_\varepsilon$. Since $u_n = t_n w_n$, $u_n \to u \neq 0$ in $\mathcal{H}_\varepsilon$ and $\|w_n\|_\varepsilon = 1$, we deduce that (t_n) is bounded in $(0, \infty)$ and that $t_n \to t > 0$. On the other hand, by using the boundedness of (w_n), we can find w such that $w_n \rightharpoonup w$ in $\mathcal{H}_\varepsilon$. Therefore, combining $t_n \to t \neq 0$ and $u_n \to u$ in $\mathcal{H}_\varepsilon$, we get $w_n \to w$ in $\mathcal{H}_\varepsilon$ and $w = \frac{u}{t}$.

(7) At pag. 246, in the statement of Proposition 6.4.14, it is needed to add "$\mathcal{J}'_\varepsilon(u_n) \to 0$". Moreover, at the beginning of the proof, one has to replace "Since $\mathcal{J}_\varepsilon(u_n) \to c$ and $\mathcal{J}'_\varepsilon(u_n) = 0$" by "Since $\mathcal{J}_\varepsilon(u_n) \to c$ and $\mathcal{J}'_\varepsilon(u_n) \to 0$".

(8) at page 271, replace "there exists a positive constant $C > 0$" by "there exists a positive constant $K_1 > 0$".

(9) at page 537, in formula (16.4.2), replace "$\frac{\partial h^u}{\partial s}(t,\tau)$" by "$\frac{\partial h^u}{\partial \tau}(t,\tau)$".

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