

## Lezioni 1 – 2

### Disequazioni razionali.

#### IN AULA.

##### A) Elementari.

a. Primo grado (il problema è: “tra tutti i numeri reali trovare quelli che rendono l'uguaglianza vera”)

$$1. \quad 2x - 3 \leq 5x + 2 \quad S = \left\{ x \geq -\frac{5}{3} \right\}$$

$$2. \quad (x-2)(x+3) > (x-4)^2 \quad S = \left\{ x > \frac{22}{9} \right\}$$

$$3. \quad 2x - 1 > 3x - (x - 2) \quad S = \emptyset$$

$$4. \quad x^2 - 1 > x^2 - 3 \quad S = \mathbb{R}$$

$$5. \quad 2 > 1 \quad S = \mathbb{R}$$

b. Secondo grado.

$$1. \quad x^2 + 4 > 0 \quad S = \mathbb{R}$$

$$2. \quad x^2 + 5x + 6 < 0 \quad S = \{x \in \mathbb{R} \mid -3 < x < -2\}$$

$$3. \quad x^2 - x \geq 0 \quad S = \{x \in \mathbb{R} \mid x \leq 0 \vee x \geq 1\}$$

$$4. \quad 4x^2 + 4x + 1 \leq 0 \quad S = \left\{ -\frac{1}{2} \right\}$$

$$5. \quad x^2 - 10x + 32 < 0 \quad S = \emptyset$$

$$6. \quad -x^2 + 2x - 1 \geq 0 \quad S = \{1\}$$

$$7. \quad \begin{cases} 2x^2 - x - 1 > 0 \\ x^2 - 2x + 2 < 0 \end{cases} \quad S = \emptyset$$

##### B) Razionali.

a. Razionali intere (prodotti di grado  $n \geq 2$  )

$$1. \quad (2x-1)^2(x+2)(-3x+1) < 0 \quad S = \left\{ \left( x \neq \frac{1}{2} \wedge x > \frac{1}{3} \right) \vee x < -2 \right\}$$

$$2. \quad x^4 - 3x^2 + 2 < 0 \quad S = \{ -\sqrt{2} < x < -1 \vee 1 < x < \sqrt{2} \}$$

$$3. \quad x^4 - x^2 < 0 \quad S = \{ -1 < x < 1 \wedge x \neq 0 \}$$

$$4. \quad 5(x-1)(x^2-6x+8) < 0 \quad S = \{ x < 1 \vee 2 < x < 4 \}$$

$$5. \quad 3(x^3-1) < 7x(x-1)$$

$$S = \{x < 1\}$$

$$6. \quad x^3 - 3x^2 - x + 3 > 0$$

$$S = \{-1 < x < 1 \vee x > 3\}$$

$$7. \quad x^3 + x^2 - 10x + 8 < 0$$

$$S = \{x < -4 \vee 1 < x < 2\}$$

b. Razionali fratte.

$$1. \quad \frac{\frac{1}{2}x^2 - 4}{x^2 + 1} < 0$$

$$S = \{-2\sqrt{2} < x < 2\sqrt{2}\}$$

$$2. \quad \frac{x-2}{x^2-4} \leq 0$$

$$S = \{x < -2\}$$

$$3. \quad \frac{x-1}{x^2-2} > 0$$

$$S = \{-\sqrt{2} < x < 1 \vee x > \sqrt{2}\}$$

$$4. \quad x + 1 + \frac{1}{x+1} > 0$$

$$S = \{x > -1\}$$

$$5. \quad \begin{cases} \frac{x^2-4}{5-x} < 0 \\ x^2-2x \leq 0 \end{cases}$$

$$S = \{0 \leq x < 2\}$$

$$6. \quad \begin{cases} \frac{2-x}{1+2x} \geq 0 \\ \frac{x}{4-3x} < 0 \end{cases}$$

$$S = \left\{ -\frac{1}{2} < x < 0 \vee \frac{4}{3} < x \leq 2 \right\}$$

## PROPOSTE lezioni 1-2.

Risolvere le seguenti disequazioni.

1.  $(x+1)(5+3x)(2x-1) < 0$   $S = \left\{ x < -\frac{5}{3} \vee -1 < x < \frac{1}{2} \right\}$
2.  $(x^2-1)(1-4x^2) > 0$   $S = \left\{ -1 < x < -\frac{1}{2} \vee \frac{1}{2} < x < 1 \right\}$
3.  $(x^2-3x-4)(x^2-25) < 0$   $S = \{-5 < x < -1 \vee 4 < x < 5\}$
4.  $x^3 - 8 \geq 0$   $S = \{x > 2\}$
5.  $x^3 - 6x^2 + 11x - 6 > 0$   $S = \{1 < x < 2 \vee x > 3\}$
6.  $\frac{1}{x^2-2} + \frac{1}{x^2} < \frac{3}{4}$   $S = \left\{ x < -2 \vee -\sqrt{2} < x < -\sqrt{\frac{2}{3}} \vee \sqrt{\frac{2}{3}} < x < \sqrt{2} \vee x > 2 \right\}$
7.  $\frac{2}{1-x^2} + \frac{6}{x^4-1} > 1 - \frac{3}{x^2+1}$   $S = \{-\sqrt{2} < x < -1 \vee 1 < x < \sqrt{2}\}$
8.  $\begin{cases} 4x^3 - 13x^2 - 13x + 4 \geq 0 \\ x^4 - x^3 - x^2 - x - 2 \geq 0 \end{cases}$   $S = \{x = -1 \vee x \geq 4\}$
9.  $\begin{cases} x^4 - x^3 \geq x^2 + x + 2 \\ \frac{x+1}{x-2} \leq 0 \end{cases}$   $S = \{x = -1\}$
10.  $\begin{cases} x^2 + \frac{1}{x^2} > 2 \\ \frac{2-x}{3} < \frac{1}{x^2} \end{cases}$   $S = \{x > -1 \wedge x \neq 0 \wedge x \neq 1\}$